Cosmology 2 (Prof. Rennan Barkana): Homework 2
Jan. 9'th, 2023; Due Jan. 30'th, 2023

Note: You are allowed to use any programming language or a program such as Mathematica, Matlab, or SageMath.

In this question on spiral structure in galaxies, assume as the underlying, axisymmetric disk, a Mestel disk, which has a surface density:

$$\Sigma(R) = \frac{\Sigma_0 R_0}{R} \ .$$

a. Find $\Omega(R)$ and $\kappa(R)$ for the Mestel disk (Note: the circular velocity depends only on the disk mass enclosed within a given radius). Use this to find the radii of the corotation and Lindblad resonances for a spiral wave solution, as a function of the frequency ω and the number of arms m.

[30 points]

b. Assume $v_c = 50$ km/s, $c_S = 12$ km/s, $m = 3$, and $\omega = v_c/(10\,\mathrm{kpc})$. Solve the dispersion relation for $k(R)$, assuming k is positive. Note: There are two solutions for k; use the one that satisfies the tight-winding approximation more accurately.

Go from $k(R)$ to the shape function, and then make a plot that shows the shape of the spiral arms in two dimensions (Hint: You may want to use a parametric plotting routine). Use different colors or line types for the different arms.

Now use one other combination of parameter values of your choice (please do not coordinate with others in the class), with some $m > 3$, and again plot the shape of the spiral arms.

[70 points]