Cosmology 2 (Prof. Rennan Barkana): Homework 1
Dec. 4'th, 2022; Due Wednesday, Dec. 21'st, 2022

General note: You may perform analytical or numerical integrations, using the software of your choice.

1. Consider a **flat** Robertson - Walker universe containing nonrelativistic matter, radiation, and a cosmological constant. List the values of the cosmic age $t(a)$ and the comoving time $\tau(a)$, at matter-radiation equality, at redshift 0, and at one other redshift of your choice (do not coordinate this choice with other students in the class). Do all this for two cases: With a cosmological constant, or without. In both cases, set $\Omega_r = 9.07 \times 10^{-5}$, and $H_0 = 67.7$ km s^{-1} Mpc^{-1}. In the first case assume also that $\Omega_\Lambda = 0.689$ (all these values refer to today).

[30 points]

2. Prove in detail the following statement:

$$\sigma^2 = \frac{1}{(2\pi)^3} \int d^3k \, \tilde{W}^2(k) P(k) \ ,$$

where σ^2 is the variance of the density field δ smoothed with a window function $W(r)$, and with $r = |\vec{x}|$ we define

$$\tilde{W}(k) \equiv \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} W(r) \ .$$

[30 points]

3. In this question on galaxy formation, assume a broken power-law as an approximate model for the standard cold dark matter power spectrum: $P(k) \propto k$ for $k < k_{\rm eq}$ and $P(k) \propto k^{-3}$ for $k > k_{\rm eq}$, where $k_{\rm eq}^{-1} = 15.3$ Mpc. Set $H = 67.7$ km s^{-1} Mpc^{-1} today. Note: It is best to split the integrals below into $k < k_{\rm eq}$ and $k > k_{\rm eq}$.

Calculate $\sigma(R)$, the variance of the mean density in a sphere of comoving radius R. Normalize the power spectrum so that $\sigma(R_0) = 0.81$ where $R_0 = 8h^{-1}$ Mpc, and then plot the normalized $P(k)$ and, in a separate plot, $\sigma(R)$. Notes: In your plots, cover the ranges of k and R that are relevant to the wide range of galaxies and of large-scale structure in the universe. Plot log-log if this makes the plot clearer.

[40 points]